Modal coherence of the 22 indian shrutis

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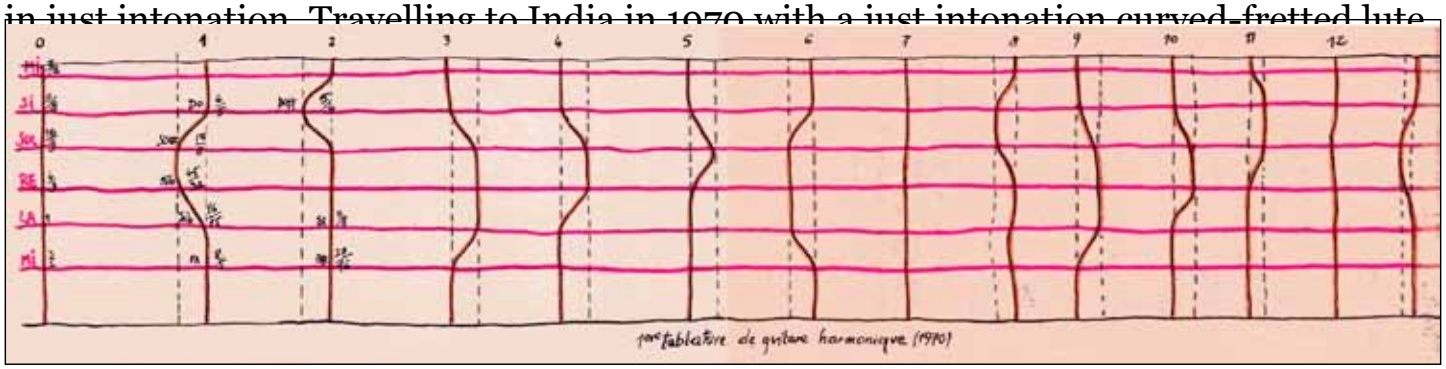
Indian music pitches are a difficult thing to measure, since in traditional râga practices musicians do not play and sing only on consonances, but also turn around and between some of them, according to specific inflexions or "*gamakas*". On the other hand, while possibly showing slight schismic variations in its definitions, the 22 shrutis pitch theory stands today as one reliable simple model, in 5-harmonic limit commonly shared by many scholars and applicable to all indian music styles, whether from South or North and from popular to classical schools. Such valuable shruti charts can be found in Professor Sambamoorthy's very complete books about Carnatic music, and other writings.
However, the acoustical basis of the shrutis can always be analysed and questionned. Ustad Ali Akbar Khan himself, who created a very high indian music school in the US made a joke by saying Hindustani râgas uses "*23 and 1/2 shrutis*"... Which somehow, as we shall see, can make sense and has the merit to leave open all listening reflexions.
Having modelled and praticed the intonation of the indian râgas myself under the ear and knowledge of my singing master Prof. M.R. Gautam, then with the help of perfect pitch instruments, my experiences arrive to this conclusion that several microtonal variations of the 5-limit classical shrutis can make sense, by the integration of two important chromatic factors, which are harmonics 17 and 19. This article explains why, and how.

I always have been interested in the indian shrutis.
It started for me before I heard about any shruti theory, by refretting string instruments in just intonation. Travelling to India in 1970 with a just intonation curved-fretted lute



this gave me the opportunity to meet, in Varanasi, my indian master, Professor Shri M.R.Gautam, director of the vocal school in Benarès Hindu University, who happened to see that instrument and was curious to meet his author.
Knowing my interest for intonation, he introduced me to the BHU musicology department and this is how, after finding what seemed to me mistakes in the 22 shrutis chart posted in a classroom, I started to draw my own chart of the 22 shrutis, from what I was hearing in the music concerts, and which were validated by my Gurujee.
This is how we then discovered, in vol.4 and 5 of the volumes of Prof. Sambamoorthy's Encyclopedia of South Indian music found in my teacher's home, that my "22 shrutis mandala" (cover image) and its ratios were in fact entirely in keeping with the South Indian shruti theory.
After 4 years spent in India, I returned to France where I fretted different guitars in alternative tunings and invented several string instruments, the chandravina, the cithararque, the aquavina, and the "dulcevina", a long strings dulcimer fretted to the 22 indian shrutis :

And I presented the 22 shrutis scale in a series of articles entitled "*Le Mandala des 22 shrutis,*

ou la rose alchimique musicale" in the traditional music review "L'Escargot Folk" (*1).
Then it was some years later, in the 80's, while creating numerous "*photosonic disks*" (*2) modelling Indian râgas, that I could experiment, in perfect intonation, the utility of extending factors 3 and 5 to 17, and 19.
The restitution of some "chromatic" râgas such as Bhairav, Shri, or Todi, etc. in pure 5-limit on a photosonic disk, demands quite high numbers of waveforms repetitions, while 19-harmonic limit often allows much simpler numbers. In addition, modelizing the scales of these râgas in 19-limit was adding alternative applications, such as intermodulations, generating difference tones, and more timbres and scales adequations.
The results of this study were compiled in the first paper I wrote in 1996, "*Modal coherence of the 22 shrutis*" (*3) which was sent to the Alain Danielou Foundation, and resumed in the Semantic instrument manual and the "Coherent shrutis" scala file (*4).
This 12 notes tuning of the first version of the *Semantic Danielou-53* instrument was covering 12 of the 22 shrutis (a chain of fifths from A to D, all modal-coherent with C or C^ if we refer to C as the tonic or "SA"), and this chromatic selection, stays in my point of view one of the best possible to tune a keyboard, or a fretted instrument limited to 12 tones, for first encounters with indian râgas, such as *Bhairavi* and several other *That râgas*.
The full 22 -c *(= abbreviation for "differentially coherent")* shrutis of the SD-53 second version completes this tuning, in a alternative to the classical 5-limit 22 shrutis already proposed in the first version of the SD-53, but making it fully "modal-coherent".
It will be included in the 2nd version of the Semantic Danielou-53, along with many new tunings and specific râgas modellings.

Let us precise first what means "*modal coherence*" : this expression is a reduction for "differential coherence of a set of notes in regard to a single tonic", which is the acoustic property to have the difference tone of each note of a mode with its tonic, to belong to this set of notes.
By default of this "modal" precision, differential coherence would apply to all intervals between any 2 notes of a scale, from smaller intervals (seconds) to larger and larger intervals (thirds, fourths, fifths, etc.), and would suggest different microtunings.
As a reminder, the general principle of differential coherence can be resumed that way : If f1 and f2 belong to a given set of frequencies and we find that 2^n(f2 - f1) = f is a member of this set, then the interval f1:f2 is deemed to be "coherent".
For a more complete explanation of the difference tones phenomena and their musical applications, refer to the article "*Differential Coherence : experimenting with new areas of consonance*" (*5).
Equal-tempered scales, and most of tempered scales other than with specific -c generators do not have these acoustic features. Some rare edos may approach these properties, but it will always be approximative.
Most JI scales generally do not have it completely either, unless some of their intervals have been microtonally retuned to be differentially coherent, whether at a modal or a scalar level, which is what I often observed with traditional scales.

In the indian shrutis classical theory, we find that 13 out of them are naturally "modal-coherent" among the 22 - which is not bad, but this also means that 9 of them are non-coherent, for any of their commonly known 5-limit ratios.

Nine are “auto-coherent” and constitute the invariable -c squeleton of the 22 shrutis : 16/15, 9/8, 5/4, 4/3, 3/2, 8/5, 16/9, 15/8, 2/1 (note that for higher shrutis the 2nd order difference tone f = 2 - [n/d] becomes more pertinent than the 1st order f = [n/d] - 1) ; and four shrutis are -c among the other ones : 10/9, 6/5, 5/3, 9/5 (for example : 6/5 - 1/1 = 1/5, which belongs to the precedent auto-coherents).

So nine shrutis are non-coherent within the other ones, which are 256:243 (same thing with its schismic variant 135/128), 32/27, 81/64, 27/20, 45/32, 729/512 (and its schismic variant 64/45), 128/81, 27/16, 243/128.
For example, 256/243 - 1/1 = 13/243 which is not close to any shruti, or 32/27 - 1/1 = 5/27 which is one comma under the pure fifth PA therefore not a shruti, etc.
It is interesting to observe the difference tones of different JI limmas (or chromatic tones), used in various musical cultures (I transpose here these difference tones between 1/1 and 2/1), among which the Safisma limma 59/56 :

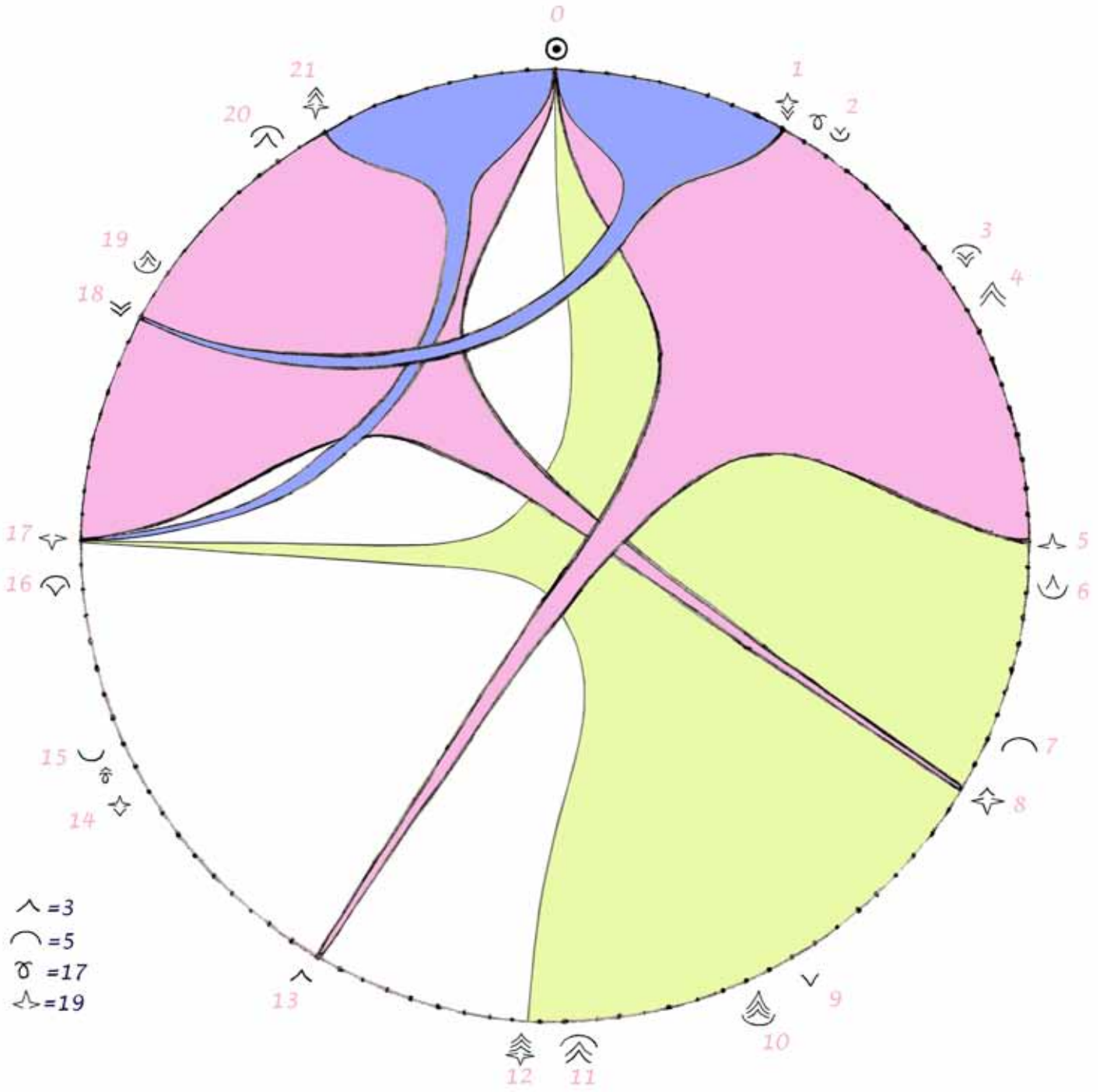
20/19	256/243	59/56	135/128	19/18	169/160	18/17
32/19	416/243	12/7	7/4	16/9	9/5	32/17

We find a large spectrum of values (one whole tone) for the limma (or “pûrna shruti”) difference tones, which are therefore very precise indicators for this interval.

In any music using a drone (such as produced by a shrutibox or a tampura in indian music), the consonances between each note and the tonic SA = 1/1 are the prioritary tonal relations to be considered. “Modal differential coherence” is therefore a pertinent aspect of the most useful scalar coherences, but however not exclusive either of the global coherences to be found between other notes than SA, such as, in indian ragas, the coherence of each note with the vadi and samvadi or each main note of a râga. We should not forget also that some pentatonic modes, for example, are said, or could prove to have “invisible” important notes, not played but very present by their harmony.

The method I used to arrive to these 22 coherent shrutis was very empirical : I checked all the closest harmonic alternatives to to each of the 9 non-coherent shrutis, a total of 50 intervals that I classed by size, then selected the ones whose difference tones were in tuned with some shrutis at some octave lower, but also which harmony made sense within most known râgas, for the pertinence of their context. What’s quite interesting, is that this arrives to a structure non-limited to modal coherence, which also makes sense in the context of râgas, as well as in general chromatic contexts.
The four naturally -c shrutis have their triads easily found in many râgas ; for example, 2/1 - 9/5 = 1/5 (upper sa - high commal-ni generates a commal-dha of 8/5) which evokes râg Darbari Kanada, etc.
We observe also that in indian music practice, when specific gamakas are used to express microtonal polarities between close shrutis, those inflexions produce generally subtle melodies in their difference tones, when these double notes are coherent within the râga.

The nine non-coherent shrutis have non-coherent difference tones with 1/1 or 2/1, that can be retuned to true shrutis without any noticable loss of their normal pitch recognition. By raising 32/27 for example by only a 513/512 schisma it becomes a very natural 19/16 whose difference tone is a pure PA, already present in the tampura and perfectly consonant with 1/1.

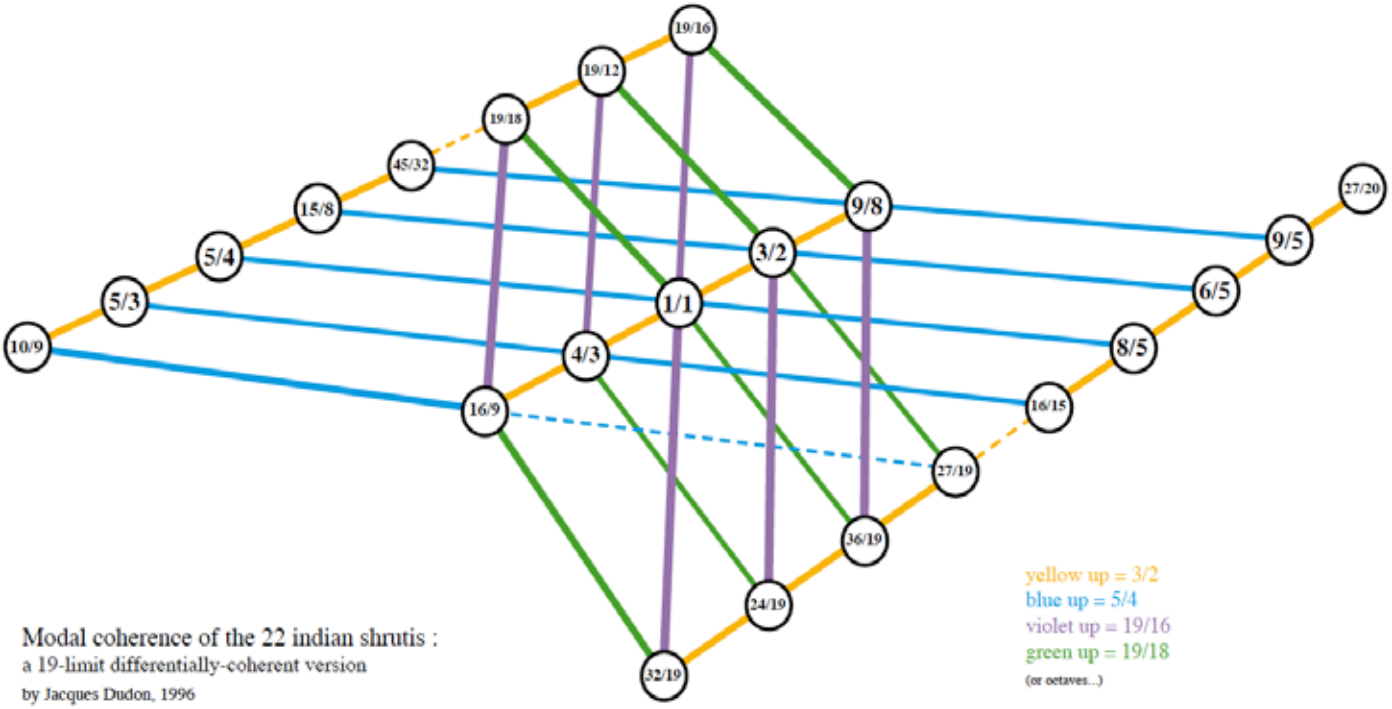


This octave wheel shows the whole coherent new 22 shruti set and represents the differential coherence of some of their L-19 intervals in the form of three types of colored -c phylacteries : 19/18 (in blue), 19/16 (pink), 27/19 (yellow-green), whose arrow indicates their difference tone position within the octave.
We note that the 19/18 “pûrna shrutis” generate a difference tone one pure major tone below them : 19/18 - 18/18= 1/18, in other words 19 - 18 = 1.
And the 27/19 higher tritone 27/19 interval restablishes the -c (27 - 19 = 8...) with the higher Dha 17th shruti.
The symbols expressing the shrutis ratios are based on the combination of 4 archetypal forms representing the factors 3, 5, 17, 19 whether upright or reversed, which in alternative to decimal numbers indicates instantly the harmonic path from 1/1 to these shrutis.

Each of the non-coherent shrutis (in 5-limit) has its own particularities ; it could be said that 27/16 is “half-coherent”, since if 27 - 16 = 11, 32 - 27 = 5 generates a very basic

shruti ; however this interval between 27 and 5 is dissonant, while tuning 27/16 to 32/19 (smaller of a 513/512) again makes the whole chord [Ga Dha Sa^] much more consonant; so 27/16 can be replaced avantageously by 32/19, whose 2nd order difference tone is another of the new coherent shrutis, 24/19 (replacing itself 81/64), and one pure fifth above.
For 27/20, which I consider the rarest of shrutis as I rarely hear singing it in this particular place, no correct alternative can be found but since its natural difference tone 7/5 stands as the best consonant substitute to 45/32, and also because of strong -c relations with PA (3/2 - 27/20 = 3/20 = commal ga) , I kept this shruti untouched.
Same thing for 45/32, non-coherent since its 1st order difference tone is 13/32, but whose 2nd order difference tone 2/1 - 45/32 = 19/32 is shruti-coherent, and because of the perfect auto-coherence of 45/32 with 3/2 and 5/4 (both produced by the tampura), the utility to retune 45/32 is not pertinent.

Thus we note that the beginning and end of the fifth chain, as well as the middle of the chain, remain identical to the classical shrutis.
The following vectorial representation resumes the features of this chain of fifths, all pure 3/2 except two 256/171 (lower by 513/512) and two 608/405 (higher by 1216/1215). All major thirds are pure 5/4, except one 304/243, raised by 1216/1215 :



A quite satisfying result, while I didn't look for it, is the final number of only 4 different intervals sizes between these shrutis (their number in 5-limit was 5...), which shows an important degree of circularity between these coherent shrutis.
Another improved feature here is that all of these 4 are epimoric : 18:19, 95:96, 24:25, 80:81.
The full octave is then composed of (19/18)^7 * (96/95)^7 * (25/24)^5 * (81/80)^3 = 2.
While the 5 lagus (25/24) have been left untouched, in dimension as well as in position, we notice of course the omnipresence of the magical ingredient of this scale, the factor 19.

Indeed, one main result of this study is to show the utility of this prime in indian music, whether in major or minor harmonies, to improve the differential coherence of the 22 shrutis, in its modal aspect, and not only limited to modal.
This is attained here by replacing, whenever possible, the 3 and 5-limit non-coherent double “pûrna shrutis” 256/243 and 135/128, by one unique 19-limit limma, 19/18.
On the contrary to a classic 5-limit shruti chart, wherever it would be difficult to say if the 1st shruti should be 256/243 or 135/128, here 19/18 stands for both.
Because dissolving the L-5 schisma 32805 / 32768 it constitues, certainly, a harmonic solution for a schismatic temperament.

About tempering the 22 shrutis

But the modelling of these 22 -c shrutis in just intonation produces an unequal schismatic sequence, which by using otherwise a single fifth or fourth tempered generator would make equal the commas 81/80 and 96/95 of the JI version.
The 19/18 limma can be seen as a rational approximation of the differential coherence [19 - 18 = 1] that would be applied everywhere in a schismatic chain, a property of the fractal “Mysteric” polynomial 8x^5 = x^2 + 32, resolved by a fourth ratio converging towards 1.33386495877 (or a fifth convergence of 1.49940215975).
Knowing that this precise generator would create an endless sequence verifying the same fractal property will help define optimal JI rationalisations for such a schismatic sequence.
But apart for making the two commas 81/80 and 96/95 equal, or extending the 21 fifths chain to a higher number (such as 53, 65, 77, or 113 notes per octave according to different generators), nothing really justifies the tempering of this coherent shrutis scale.
On the contrary, this precise JI sequence allows a plurality of differential coherence properties : in just intonation 19 restablishes also the “Melkis” differential coherence of the smaller minor third interval (Bhairavi commal ga described as 32/27 in the classical shruti nomenclatura), replaced here whenever possible by a 19/16, whose difference tone is improved to a pure panchama shruti.
On the contrary, an equivalent tempered schismatic constant generation would not make possible to have both exact -c of the limma 19/18 and the minor third 19/16 :

19-18 = 1, or 8 v^5 = v^2 + 32, “Mysteric”, is achieved by v = 1.33386495877

19-16 = 3, or 2v^4 = 4v + 1, “Melkis” is achieved by v = 1.33415744713013

Both would result in quite different but good tempered versions of the 22 shrutis, while requiring a choice which is not necessary in just intonation.
Just in case, for a tempered version the sequence providing the -c of the limma 19/18 seems to be more central to a classic schismatic tuning, producing major thirds closer to 5/4.
Optimal values for other tempered schismatic generators should then be close to what makes the limma 19/18 as pure as possible (a pure 19/18 is attained with a fifth of 1.499414743, while its perfect -c in a fourth or fifth chain is more precisely achieved with 1.49940215975).
Additional 17-limit shrutis

While all raga families can be expressed successfully with the present 22 shrutis, Râga Bhairav, said to be the king of Râgas and incarnation of the yogi Shiva, has a special sadhja - commal re - suddha ga harmonic feature that makes an alternative commal re even more coherent, with a ratio of 17/16. This starts with 17 - 16 = 1 which gives to this shruti a perfect steadiness, same thing with 20 - 17 = 3 which confirms its coherence with the harmonic major third, and also with the tritone 17 - 12 = 5 and the larger tone around SA 17 - 15 = 2, both coherent again with the same shrutis.
Since adding two additional shrutis to this 22 notes set makes a tuning file whose mapping fits into 2 octaves of 12 notes of an ordinary keyboard, 17/16 and its counterpart in the higher tetrachord 51/32 thus provide essential -c complements for the higher commal shrutis re & dha (#2 & 15), in two sensible areas of the 22 shrutis regular sequence.
This is the scala file of this augmented 24 shrutis scale :

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! coherent_shrutis-22.scl
!
Indian Shrutis, modal -c with C or C^
24
!
 19/18        ! -5          ! 95:96
 17/16        !
 16/15        ! +7          ! 24:25
 10/9         ! -10         ! 80:81
 9/8          ! +2          ! 18:19
 19/16        ! -3          ! 95:96
 6/5          ! +9          ! 24:25
 5/4          ! -8          ! 95:96
 24/19        ! +4          ! 18:19
 4/3          ! -1          ! 80:81
 27/20        ! +11         ! 24:25
 45/32        ! -6          ! 95:96
 27/19        ! +6          ! 18:19
 3/2          ! +1          ! 18:19
 19/12        ! -4          ! 95:96
 51/32        !
 8/5          ! +8          ! 24:25
 5/3          ! -9          ! 95:96
 32/19        ! +3          ! 18:19
 16/9         ! -2          ! 80:81
 9/5          ! +10         ! 24:25
 15/8         ! -7          ! 95:96
 36/19        ! +5          ! 18:19
 2/1          ! 0           ! 18:19
!
! The middle column indicates, for the 22 shrutis regular sequence,
! the order of fifths in the schismatic chain, from D- to F+
! which replaces both 135/128 and 256/243 by the 19-limit limma 19/18,
! thus dissolving, in addition to the 5-l. schisma, 1216/1215 and 513/512.
! The right column indicates the intervals between a shruti and the next one :
! these are of 4 types, all epimoric : the limma 19/18, the lagu 25/24,
! and two commas : the pramana shruti 81/80 and the 19-limit comma 96/95.
! (19/18)^7 * (96/95)^7 * (25/24)^5 * (81/80)^3 = 2
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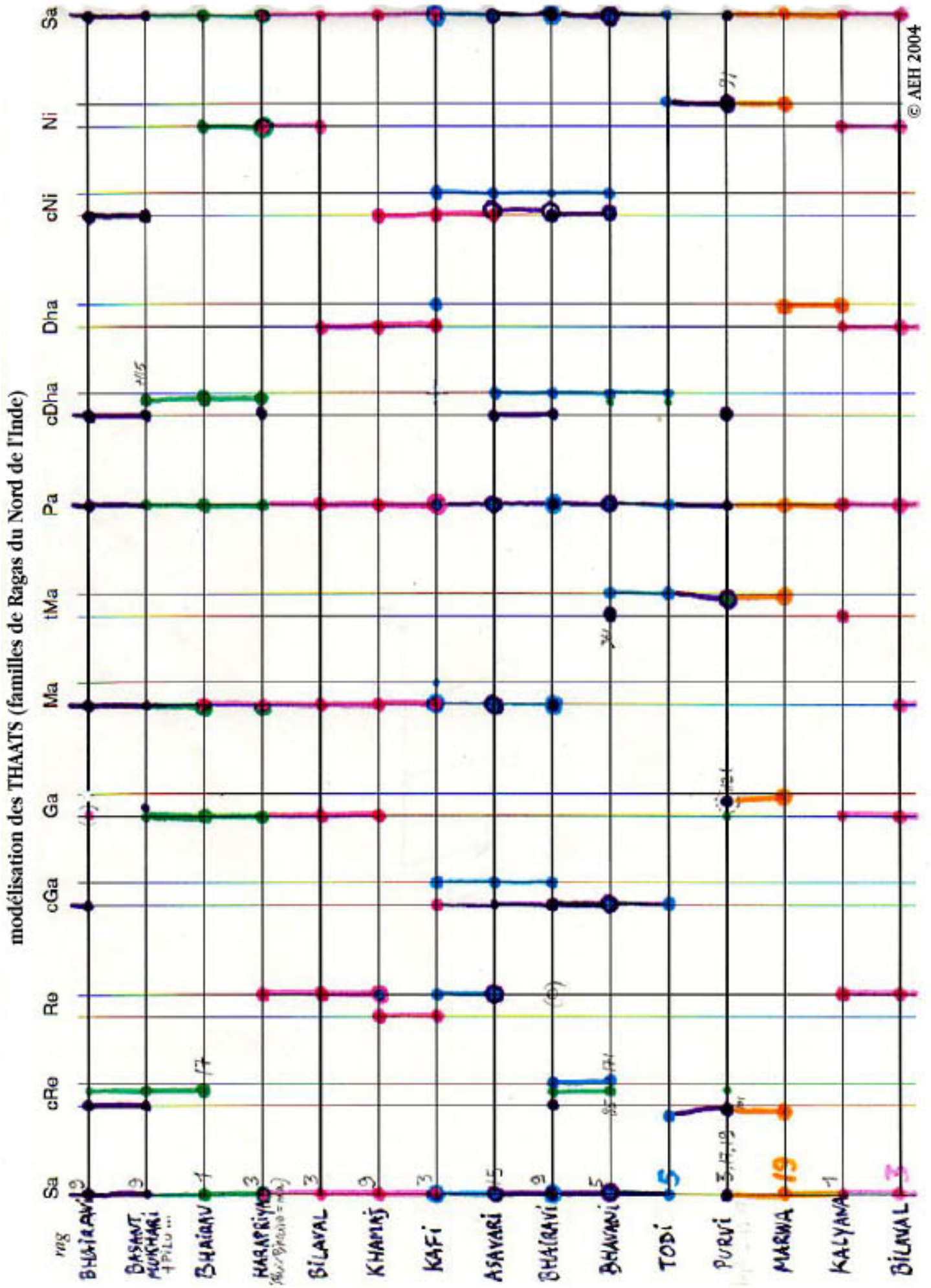
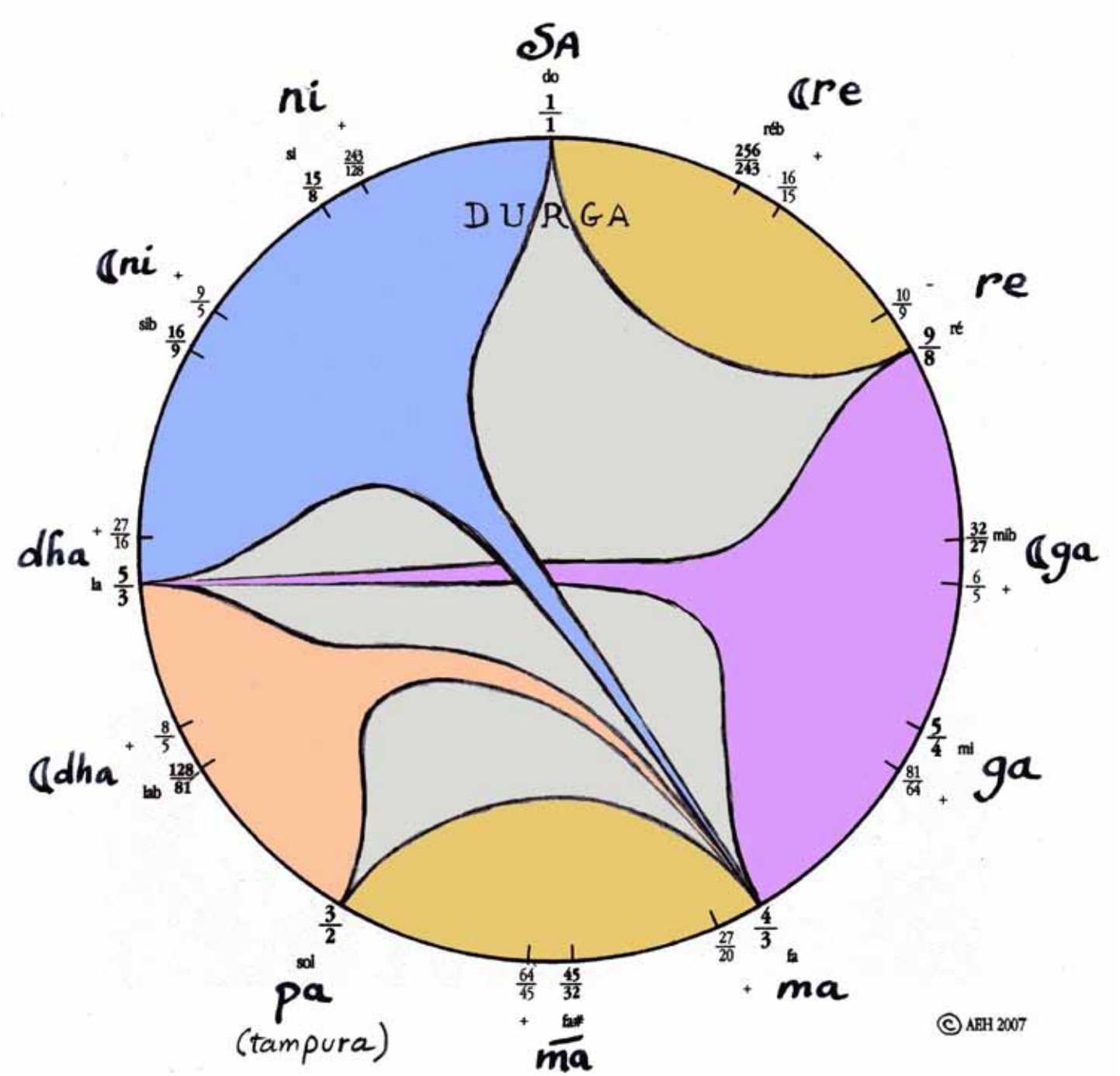
(annex : shruti modelling of the Thât râgas and a few others)



Classical differential coherence for a 5-limit pentatonic frame : râga Durga. Applying a *Murchana* (transposition) in different shrutis this basic structure is found in many râgas.



Notes :

*1. L'Escargot Folk, 1974, # 32,33, 34, 37
*2. jim.afim-asso.org/jim2002/articles/L12_Arfib.pdf
Lumières audibles, video by Dana Sardet : https://vimeo.com/676062581
*3. "Modal coherence of the 22 indian shrutis", Jacques Dudon/AEH 1996, augmented edition AEH 2020.
*4. This instrument is downloadable free, as well as its manual and its scala files.
*5. That can be found in the album ""Differential coherence"of the FB "Intonation juste" group : https://www.facebook.com/media/set/?set=oa.542824646049240&type=3 or downloaded through Academia's website.