Seven-limit Slendro Mutations

by Jacques Dudon

We should not be surprised that traditional musics may be based on Just Intonation models, as consonances have always provided the simplest references for tuning by ear. Even though the inharmonics of some instruments may alter our pitch perceptions, Just Intonation remains the most reproducible and universal solution. Often, different consonances give rise to closely spaced tones; these phenomena encourage musicians to further refine their intonations. A taste for certain ambiguous pitches (i.e., multiple, microtonally distinct pitches hiding behind a single pitch name) is inherent in Just Intonation. Tempered systems reduce closely spaced tones to single-tone abstractions, whereas Just Intonation offers an open space containing infinite alternatives within a musical landscape always in evolution. In a few traditions, international models are consciously defined in more or less pertinent ways, but in most cases tuning procedures are transmitted aurally, and an occasional fixed-pitch instrument is passed from hand to hand. Since Western tempered music has spread to every part of the world, knowledge of intonation is more fragile than ever, and there is an urgent need to record the traditional intonations that have not yet disappeared.

While studying Javanese music in the 1970's, Lou Harrison performed a very useful work for future generations: he discovered several different types of seven-limit just slendro tunings for the several gamelan he built with William Colvig. These tunings have since been recognized by many Javanese master musicians as correct tunings for several specific styles of slendro, and they are now used by many Javanese musicians. Although they have very precise forms, the two Javanese scale types, slendro and pelog, are not mathematically codified by Javanese musicians and theorists. Harrison says his tunings are just approximations of the Javanese scales, but I have yet to hear a traditional slendro tuning that doesn't correspond to one of Harrison's tunings or, at worst, to a combination of two or three of them.

Although it is possible to create an infinity of different seven-limit pentatomics, I found that the five main slendro scales used by Lou Harrison are related by a very special property: it is possible to shift from any one of these scales to any other by adjusting only one tone. Further, Harrison's are the only seven-limit pentatonic scales that have this property. Exactly 24 such mutations are possible among the five scales, which shows the strong unity of that small slendro family.

Figure 1 (page 4) lists the five scales (labeled N, M, A, S, and J) in terms of their harmonic structures, regardless of their octave positions, the scales being octave-repeating. Javanese musicians have no names to distinguish these five tunings; those used in this article are of my own devising.

"N" stands for "natural," as this scale is found in the lowest segment of the harmonic series of any of the five. In Javanese music I have not heard this scale used in an unambiguous form; it tends to be blended with "M" (see next).

"M" stands for "Mi," or the mirror image of N. The aliquot divisions of a perfect fifth into three parts and of a perfect fourth into two parts on a stringed instrument produce this scale, which is the one Lou Harrison chose for the Mills College Gamelan he built in 1980 with Bill (text continued on page 4)
(Slendro, continued from page 1)

Colvig: it is a very popular scale in Java, where you can hear it in Bandung, Kota Gede, Jakarta, Banju Wangi, and other locations.

“A” stands for “Si Aptos,” as it is Lou Harrison’s Aptos School Gamelan tuning; it has been recognized as a typical Central Javanese tuning.

“S” stands for Surakarta and “J” for Jogjakarta, as these last two tunings are strongly established in the court traditions of these two central Javanese cities.

Actually, it is probable that these five scales are used in many other equatorial cultures, such as in Africa, and my intuition tells me they are closely connected with the very complex Mbuti Pygmy music. I recently met a balafon (African xylophone) maker and musician from the state of Burkina Faso, who presented me with a balafon supposed to be tuned in his country’s principal pentatonic tuning, which I could clearly identify as a perfect Surakarta slendro.

Figure 2 shows the successive intervals of the scales in Figure 1, using the same modes, except in the case of J, which was shifted by one tone so as to share three common tones, in the form of a 21:24:28 tetrachord, with the other four scales. This is done to reveal the simplest transitions between scales. (The same could have been done by aligning the 6:7:8 tetrachords that are also present in all five scales.)

8:7 is certainly the predominant interval here; it occurs twelve times, followed in order of frequency by 7:6 (eight times, present at least once in every scale), 9:8 (three times, generating double-tetrachord structures in N, M, and A), and the only non-superparticular, 147:128, which occurs twice. This interval, the closest in size to a division of the octave into five equal steps, is also one of the rarest. No consecutive 7:6’s or 7:6-147:128 pairs are found among the five scales. These intervals would generate other consonant, but less circular scales, which would not mutate into S or J by a single-tone inflection.

In Figure 1, the mutation between S and J involves a shift of one tone by a 49:48 “large comma” interval (35.5 cents); here, the SJ mutation is of a softer type, using a shift of 1029:1024 (8.4 cents or about one-third of a comma). As with N and M, S and J are compliments; this makes the 1029:1024 interval especially important—shifting a single tone by this interval changes one’s perception of the entire scale, from a harmonic series segment to a subharmonic series segment. This also changes the perceived tonic of the scale—since S is the harmonic-series version, its fundamental has more weight than that of J. In the Javanese court music traditions of the two cities, these two scales, instead of being considered the same (which could well have been excused, considering the average pitch precision of the gongs), are developed in quite distinct modes (if we can employ this term in the Javanese context), which also greatly simplifies their recognition. The 147:128 interval, the remainder when two 8:7’s are subtracted from a 3:2, has an interesting ambiguity, due to its closeness in size to the septimal second 8:7 and to its position in the S and J scales, which suggests a minor-third feeling. It is rather surprising that such a small interval as 1029:1024 should give rise to the sensation of a semitone when it occurs as the shift between the septimal second (8:7) and the “minor third” (147:128). The shifts involving the larger intervals 64:63 and 49:48 are all the more blatant.

Figure 3 shows the fourth (or hyper-major third) intervals of the five scales; 4:3 is in the majority in N and M, but is in a minority in A, S, and J, which use less consonant fourths and thirds. It is important to know that in Indonesian modes it is not required that the effective tonic of a scale be accompanied by a 3:2 perfect fifth; for example, in its usual tonality, the Surakarta Solo style
uses a wonderful 32:21 \((8:7 \times 8:7 \times 7:6)\) of 729 cents over the tonic, and the scale has two of these, plus a 49:32 (738 cents). I guess it just “dances with the wolves,” as we might say. [You might say that, Jacques, but I, for one, wouldn’t dare—Ed.] As there are four types of “seconds” in the five scales, we find here that there are also four types of “fourths.” As shown in Figure 4, the relative sizes of both the four seconds and the four fourths differ by 28:27, divided as 64:63–1029:1024–64:63. Happy winners are 4:3 (twelve times), 21:16 (eight times), 64:49 (three times) and 9:7 (twice). The 49:36 type of fourth, larger than 4:3, does not occur in any of these scales. As in any anhemitone-type pentatonic scale, all of these scales have a single “major third” (9:7 or 64:49), while the subtle polarity interval 1029:1024 (between 64:49 and 21:16) appears again to be the borderline between the “major thirds” and the definitively fourth-type intervals, 21:16 and 4:3.

After this rather linear analysis, I thought that a threedimensional object would be best suited to offer a more synthetic vision of how the five scales were related. The “Slendro Pentahedron” of Figure 5 is designed to be photocopied and glued to a piece of cardboard, then cut and assembled with glue. (Incising half-way through the cardboard along the fold lines before folding around a ruler will help produce straight edges.) This pentahedron may be used as a reminder of the five scale mutations, but it is primarily a conceptual object for meditation, which I dedicate to all gamelan tuners. There are no pentatonic scales within any limit that are both closer to five-tone equal temperament and more consonant than these five; thus this form is the last irreducible representation in space of the most circular division of the octave into five musical intervals.

Many musicologists have long cherished the belief that five-tone equal temperament is the tuning model for the slendro. I suppose they feel lost outside of any temperament. Sorry! Five-tone equal temperament will never have more than five intervals, including the octave. The slendro system is a thousand times more subtle and more consonant than that.

But let us turn the question another way: suppose you have a file, and six slabs (five plus one for the octave), that you wish to tune to five-tone equal temperament (or, to be more realistic, let us say you are a pentatonic explorer, and you had this brilliant idea of having all your successive tones separated by the “same” interval). You are probably like me in that, even if you really want five equal steps, you are unable to sing the powers series of the fifth root of two by heart—so you go on filing the slabs, until, unless you are masochistic, you find something that pleases you.

Hence, the following questions arise:

1) How can you avoid having one or more intervals in the vicinity of 8:7, which you may wish to perfect, simply because they sound better?

2) Given that 8:7 is closer to one-third of a 3:2 than to one-fifth of a 2:1, how can you avoid, if one of your seconds is surrounded by intervals slightly smaller than the average, having at least one interval in the vicinity of 3:2 that you may want to tune in for the same reason? Or how can you avoid having at least one interval in the vicinity of 4:3, if one of your seconds is surrounded by intervals slightly larger than average, i.e., in the neighborhood of 7:6?

3) And the same thing applies to the complementary 4:3 or 3:2. In other words, in searching for a division of the octave into five equal parts, the occurrence of seven-limit intervals is practically inevitable. And once a seven-limit pentatonic is accepted, even if, for some reason, the tuning gets modified, you have all the chances described above to fall onto another seven-limit pentatonic, built on the same familiar material. Unless the shifting tones go beyond the general range of the 28:27 diasis described before, you can always slip into another of the five scales, simply because that’s the way each of them is connected with the four others. Slendro is very much like “the tree that hides the forest”: move very slightly from it, or change its accents and that’s it: the tree becomes the forest again. One is often unable to decide from which angle to view it, so multi-dimensional is its modal aspect.
Figure 5. The seven-limit slendro pentahedron
Figure 6.

I guess that's probably what Lou Harrison means when he talks about the "slippery" quality of slendro.

On the pentahedron, the five scales are represented by the five vertices, which are distant from each other by lengths proportional to the logarithmic dimensions of the three intervals necessary to their corresponding mutations:

\[
\begin{align*}
\text{SJ} &= \log \frac{1029}{1024}; \\
\text{MN} &= \text{NJ} = \text{MS} = \text{AJ} = \text{AS} = \log \frac{64}{63}; \\
\text{AN} &= \text{AM} = \log \frac{49}{48}
\end{align*}
\]

\[\text{(}=\text{NS}=\text{MJ},\text{but the trapezoid NMSJ could not integrate that)};\]

This is the only possible 3D representation of the five scales, proportional to their smaller mutational intervals. The volume's plane of symmetry, passing through A, also respects the scales' harmonic symmetries, between M and N, and between S and J. The modulation ratios are indicated along the edges, the positions of the numbers indicating whether the pitch has to be raised or lowered in order to shift to the corresponding scales; for example, N 64:63 M indicates that the tone must be lowered from M to N 64:63 in going from N to M or raised from 63 to 64 in going from M to N. On each vertex, we find the harmonic structure of each scale according to the 3 and 7 coordinates, with pitch 1 circled (indicated similarly in all other figures: this tone defines the "major third" position), plus four or five letters indicating the destination and effect of the mutating tones. In order to know in each case which is the changing tone, you must refer to both scales' structures. Figure 6 shows how to interpret these symbols for the Jogya tuning. Notice the two mutations to different transpositions of scale S; one with pitch 3 shifted by 1029:1024 and the other with pitch 1 shifted by 49:48. We also see in Figure 6 a second type of single-tone mutation possible between N and M, in which pitch 1 of M is lowered by 28:27 (49:48 × 64:63), which sounds a little outside the usual slendro aesthetic.

This mutation, by the way, is found among the "forbidden tonalities" of the early baroque meantone tuning. Tuned in the conventional form, with the chain of meantone fifths running from B♭ to F♯, scale N comprises the tones G♯, B♭, C♯, E♭, and F; and scale M comprises the same tones except with F♯ in place of F. Baroque composers who used the wolf fifth and other baroque "estrangetés" in diverte piece apparently felt that they were touching something oriental, though they had probably never heard of slendro. Actually, in addition to the just major thirds that are the reason for its existence, meantone includes many interesting intervals: 32:25, a close approximation to 9:7; a wolf (G♯–E♭) that differs from 49:32 by .01 cent; three approximate 7:6's (B♭–C♯, B♭–F, and F–G♯); and two approximate 8:7's (C–E♭ and G–B♭). After the major thirds, the approximate 7:6's and 8:7's in the "hidden side" are meantone's purest intervals. Eighteenth century French composer Jean-Philippe Rameau describes the mean-tone "12:7" (augmented sixth) as "very plaintive." Was he having the seven-limit blues?

Now, the next practical question is: on a keyboard, or any other fixed-pitch instrument, how many tones do we need to experiment with the five scales together? Seven tones are sufficient for four scales (Figure 1), but eight is the minimum necessary to have them all. The eight tones of Figure 1 also generate an extra N (×7), plus two scales with consecutive 7:6's (36:42:49), and only the 49:48 mutation between S and J; the eight tones of Figure 2 have none of that, and only the soft SJ mutation.

The "circular matrix" of Figure 7 includes the eight tones of Figure 2, and is probably the best solution for experimenting with all the possible mutations without
leaving the slendro genus; with only ten tones it generates a total of ten regular scales (detailed in Figure 8), favoring S and J transpositions, plus eight anomalous scales of seven types, and slight variations of A, S, and J by one 1029:1024 (dotted lines). All scales share the same central two tones (1/4 and 9/7), separated by a septimal second.

On a twelve-tone-per-octave keyboard, it is advisable to double the 1/4 and 9/7 on consecutive keys.

**Note:**

1. Doty, David B. “The Lou Harrison Interview” 1/1 3:2, 1987 1/1